

3.22 Trigonometric Form of Complex Numbers

The complex number $a + bi$ can be thought of as an ordered pair (a, b) .

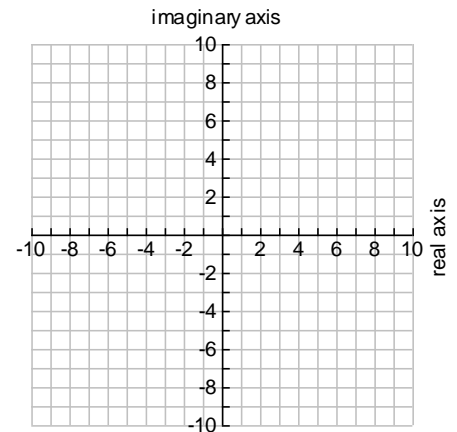
We graph it on the **complex plane** where the horizontal axis is called the **real axis** and the vertical axis is called the **imaginary axis**.

Absolute Value or Modulus: $|a + bi| = \sqrt{a^2 + b^2}$. (The distance between the number and the origin on the complex plane.)

Examples: Graph each complex number and find its absolute value.

a) $5 - i$

b) $-6 + 2i$



Trigonometric Form of a Complex Number

If $z = a + bi$ is a complex number, then the trigonometric form of z is

$$z = r(\cos \theta + i \sin \theta), \text{ sometimes abbreviated } z = r \operatorname{cis} \theta,$$

where r is called the **modulus** and θ is called the **argument**, defined as the angle in standard position whose terminal side contains the point (a, b) .

$$r = \sqrt{a^2 + b^2}$$
$$a = r \cos \theta \text{ and } b = r \sin \theta.$$

We usually use the smallest possible nonnegative angle for θ .

Examples: Write each complex number in trigonometric form. Express θ in degrees.

a) $-2\sqrt{3} + 2i$

b) $5 - 4i$

Example: Write the complex number $12\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$ in the form $a + bi$.

Product and Quotient of Complex Numbers

If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$, and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$, then

$$z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

Examples: Find the product and quotient using trigonometric form.

$$z_1 = 4\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right), \quad z_2 = 8\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

a) Find $z_1 z_2$

b) Find $\frac{z_1}{z_2}$

Complex Conjugates

The conjugate of $r(\cos(\theta) + i\sin(\theta))$ is $r(\cos(-\theta) + i\sin(-\theta))$

A complex number times its conjugate equals r^2 .

$$\begin{aligned} \text{Proof: } & r(\cos\theta + i\sin\theta) \cdot r(\cos(-\theta) + i\sin(-\theta)) \\ &= r^2(\cos(\theta - \theta) + i\sin(\theta - \theta)) \\ &= r^2(\cos 0 + i\sin 0) \\ &= r^2(1 + 0i) = r^2 \end{aligned}$$

Example: Find the product of the following and its conjugate: $6\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$.